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# EFFECT OF LASER DISTANCE MEASUREMENT FOR FATIGUE CRACK DETECTION ON ALUMINIUM PLATE USING LASER DOPPLER VIBRO-METER

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#### ABSTRACT

Fatigue cracks can occur because the material is unable to withstand the load applied repeatedly. A nonlinear vibroacoustic method was introduced to overcome this problem. This is because this method is one of the best solutions because it is suitable for detecting fatigue cracks which is sensitive enough to detect small cracks. The aim of the research is to determine the effect of laser distance measurements on fatigue crack detection using the vibroacoustic method. The vibroacoustic method is a method based on the propagation of high frequency sound waves in solid structures with low frequency excitation. The trained output signal will be converted from the time domain to the frequency domain which is supported by the use of MATLAB software. The analyzed data shows that measuring the laser distance will influence the crack detection process. Then data obtained from the power spectrum using nonlinear vibroacoustic testing is used to plot the high frequencies of 60, 150, and 200 kHz with the laser measurement range. And the research results show that the 60 kHz frequency is the best choice and is sensitive enough to detect small cracks and is able to detect relatively large cracks well. **Keywords :** Fatigue Crack, Vibro-Acoustic, Laser Doppler Vibrometer

# 1. Introduction

Vibration-based testing has been developed in previous years as a promising strategy for structural health monitoring. The use of SHM is based on vibration because the dynamic characteristics of a structure are a component of its mechanical properties. However, changes in mechanical properties due to defects will cause major changes in the dynamic vibration characteristics of the structure (Ma et al., 2022), (Roemer, 2019), (Steigmann et al., 2020). Cracks are related to material or geometric discontinuities and static and dynamic stress concentration (Tian et al., 2019), (Wang et al., 2021a), (Zhao et al., 2023). To simulate them, a numerical finite element model was built, and a plate was damaged with notches. Its stationary dynamic behaviour was measured with a Laser Doppler Vibrometer (LDV) (Wu et al., 2020), (Dharmadhikari et al., 2022).

There are many methods to detect fatigue failure. In a previous study by that at the point when any mechanical component or part is subjected to applied load, at that point there will be conceivable of fatigue failure (Jin & Shokouhi, 2022), (Dharmadhikari & Basak, 2022). The method of fatigue failure of a part is only the breaks created on their surface. The significance of an early location of crack seems, by all accounts, to be crucial for both wellbeing and any reason since fatigue crack is the potential for deleterious fatigue failure (Mrzljak et al., 2023), (Affandi et al., 2021)(Satria et al., 2022), (Nasution et al., 2020). The strategy for detecting cracks in cantilever beams or panels relies entirely on variations in the component's vibration parameters with respect to normal frequency changes. There are several damage detection methods that can be used to measure structural damage, such as acoustic emission (AE) measurements, ultrasonic testing (UT), radiography (RT), acoustic vibration temperature (IT) and infrared measurements. Unfortunately, all methods have different limitations and sometimes testing cannot be performed in certain cases (Lee et al., 2019), (Zhu et al., 2021). For the Vibro-acoustic method, the results analyzed are non-linear sound modulations. Experimental study of nonlinear acoustic vibration wave modulation in cracked beam panels (Chen et al., 2020),

(Wang et al., 2021b), (Sampath et al., 2022). The study revealed that energy dissipation – not crack opening and closing – is the main mechanism behind the nonlinear modulation. The result is a relatively weak strain field that can be used to detect cracks in metal structures. A clear relationship between sideband and crack length is also demonstrated, providing experimental evidence for the recently proposed non-linear and non-classical acoustic wave interaction mechanism (Mieloszyk, 2021). Nonlinear acoustic vibration is a very reliable and sensitive method for detecting damage. This method is based on the propagation of high frequency sound waves in solid structures with low frequency excitation. The interaction of sound waves with changes in material properties or geometry causes wave distortion effects. The so-called nonlinear sound effects are amplified by low-frequency excitation. Although it is widely accepted that nonlinear modulation is caused by crack opening/closing activity, there are still other physical mechanisms involved behind such modulation (Xiao et al., 2020), (Zhang et al., 2022), (Iqbal et al., 2023).

The most widely recognized structural defect is the presence of fatigue crack (Williams et al., 2020), (Alahnomi et al., 2019), (Dharmadhikari & Basak, 2021). Fatigue crack can show in the structures because of different reasons (Rechena et al., 2020), (Bo et al., 2023). The presence of cracks will cause changes in local stiffness and can significantly change the mechanical properties of the entire structure. Therefore, the nonlinear vibroacoustic method is very suitable for determining the presence of fatigue cracks because it is very sensitive to even minimal damage. This article presents experimental results showing the effect of laser distance measurements on fatigue crack detection using the vibroacoustic method.

#### 2. Research Methods

The material properties of the aluminum plate used in the test is AL-2024 which is high strength aluminum. It is commonly used in aircraft structural parts and accessories, hardware, and transportation industry components. The dimensions of the plate are 400 mm x 150 mm x 2 mm, and the aluminum plate used as a test object has the material properties. A hole in the center of the plate is necessary to induce cracking and lock the plate in the fatigue testing machine. Fractures were initiated with a 4.0 mm groove in the center of the plate, as shown in Figure 1. The plate was drilled to create a 2.0 mm diameter hole using a hand drill. Next, a very small groove is made using EDM Wire Cutting, as seen in Figure 2.



Fig. 1. Schematic Drawing of Plate during Modal Analysis Experiment



Fig. 2. Dimension of notch on aluminium plate

Electron discharge machining (EDM) is a manufacturing process that uses an electrical discharge or spark. The finished 6 mm notch in the aluminum sheet is shown in Figure 2. The diameter of the notch is 2.0 mm while the length of the notch is 3.0 mm. According to Figure 1, the SLDV laser will point upwards 30 mm above the fracture line. A piezoelectric transducer (PZT) was then installed 30 mm from the bottom of the crack to generate a high frequency (HF) signal excitation. The high frequencies used are 60, 150 and 200 kHz. In addition, a shaker will be installed at the corner of the plate to generate low frequencies 62.5, 109.5 and 188 Hz. The results of this experiment will be measured and recorded by SLDV.

# a. Modal Analysis Test

Modal analysis is required to determine the neutral frequency parameters (low frequency and modal shape) of the cracked plate. Physical properties such as mass, damping, and structural stiffness contribute to the shape of the mode.



Fig. 3. Experimental setup for modal analysis test

In this sense, there will be some changes in the mechanical properties if the physical properties of the structure are changed. Modal analysis involves applying excitation to a structure and finding the resonance frequency of the structure (Sohn et al., 2003). The sample setup for modal analysis includes several measurement and test sample structures, as shown in Figure 3. A Scanning Laser Doppler Vibrometer (SLDV) was used to measure the resulting vibration response.

Table 1 - Function Generator Parameter and Configuration	
Items	Characteristics of assigning values or
	parameters
Frequency	1 ~ 2000 Hz
Sweep time	2 s
Amplitude	10.0 Vpp
Sampling size	20 kS/s
Sampling	100 Ksample
Item	Parameter
Displacement	± 122.5 um
Velocity	$\pm$ 490 mm/s
Acceleration	± 156.000 g
Max. Frequency	500 kHz
Filter	Off

As showed in Figure 3 the laser pillar from the SLDV (SWIR OptoMET) was point 30 mm over the crack line to quantify the yield motion from the excitation. Function generator (Tektronix AFG 3022) will controlled the low frequency signal. Moreover, a mechanical shaker function (TIRA GmbH sort S 50018) is used to incite low frequency signal which amplified by a power amplifier (TIRA sort BAA 60). The frequency response of the system can be measured through sweep signal excitation. The mechanical shaker was attached at 25 mm x 25 mm on the base right corner edge of the plate. The vibration reaction was caught and recorded by SLDV. The parameter setup for the function generator, SLDV and power amplifier were given in Table 1 and Table 2 separately. All the parameter used was fixed for the modal analysis test.

#### b. Vibro-acoustic Analysis Technique

This test was carried out to determine the relationship between nonlinear acoustic modulation and test parameters (crack size, excitation level, and excitation vibration mode). Vibration-acoustic interactions (low frequency pump waves and high frequency probe waves) will form sidebands. Typically, this test is carried out to test the effects of nonlinear audio modulation stimulated using high and low frequencies. The board will be hung with a thick wire/spring. A mechanical stirrer is used as a low frequency generator while a piezoelectric transducer is used as a high frequency input. The excitation waveform will be generated by a function generator and a power amplifier is used to open the vibration signal. A schematic overview showing the experimental setup is shown in Figure 4.



Fig. 4. Schematic Diagram of Vibro-Acoustic Test

The vibration mode frequency is used for low frequency vibration excitation, and high frequency excitation, namely ultrasonic waves of 60, 150, 200 kHz, is used using a PZT transducer. The vibration response is then obtained using an oscilloscope whose excitation signal is generated using a waveform generator. Parameter configurations for the function generator, SLDV, and power amplifier are presented in Table 4.

# 4. Results and Discussions

Frequency response function (FRF) plots are commonly used in modal analysis to determine the maximum resonant frequency response of a structure. The vibration response is recorded and recorded by SLDV. To determine the resonant frequency, the plate will be excited with different frequencies between 1 Hz to 2000 Hz (scan). All data collected is in the time domain and frequency domain as shown in Figure 5. The vibration modes and natural frequencies or resonances can be shown in Table 4. Then, using MATLAB software, the collected data will be converted into FRF. From the analysis results, the frequency response function (FRF) is obtained as follows.

$$\Gamma xy = \text{tfestimate } (x, y) = Pxx / Pxy$$
(1)

The transfer function  $T_{xy}$  estimated using the power spectral density of the input signal vector Pxx and the power spectral density of the output signal vector  $P_{xy}$ .

No. Vibration	Natural Frequency Value (Hz)
Mode	
1	62.5
2	109.5
3	188



Fig. 5. Input Signal A) And Output Signal B) In Time Domain for Modal Analysis



Fig. 6. Modal Analysis Result for Vibration Frequency Response Mode Function On Cracked Aluminum Plate

Figure 6 shows the results of the frequency response of an aluminum plate where several vibration modes of the damaged plate can be observed. When the resonant frequency is reached it will reach a maximum. In addition, the collected data can determine the shape mode of the aluminum plate. Another recommended way is to carry out theoretical validation or simulation.



Fig. 7. Modal analysis result

From Figure 7, the first vibration mode represents a single bending mode of 62.5 Hz. The natural frequency of the second vibration mode is 109.5 Hz. The third vibration mode is 188 Hz which indicates the presence of a double bending mode. The shape of the mode depends on the shape of the object's surface and its boundary conditions. At each particular frequency, it is known that a solid structure or solid object has its own natural frequency. The structure will mostly respond to an external cyclic force excited at one of its resonant frequencies. In this research, mode shape analysis is an important step to determine the effect of laser distance measurements in identifying defects in aluminum panels.

Acoustic Vibration Analysis to measure sideband intensity, a comparison is used between the amplitude of the first sideband frequency and the amplitude of the high frequency. This ratio is called the R value and is written as follows.

$$R = \frac{R_1 + R_2}{R_0} \tag{2}$$

Where  $R_0$  is the peak of the high frequency excitation,  $R_1$  and  $R_2$  are the spectral amplitude of the first pair of sideband. Figure 8 indicate that the intensity of modulation parameter for three mode shape analysed and selected high frequency used to find R value for the laser distance measurement. The output signal data which is the R-value will be plotted through MATLAB software. The data shown is in frequency domain and will be analysed.





Figure 8 shows an example of a power spectrum obtained during acoustic vibration testing for HF excitation at 60 kHz with one of the modal excitation frequencies. From the picture it can be seen that the cracked aluminum plate causes amplitude modulation because the ultrasonic waves are modulated by low frequency oscillatory waves. Mode form frequencies of 62.5 Hz, 109.5 Hz, and 188 Hz were used for low frequency stimulation. Figure 8 also provides an example of the power spectrum at 62.5 Hz for low frequencies at a distance of 1 meter from the SLDV. The range of SLDV aluminum panels will increase from 1m to 5m with 3 selected high frequencies namely 60, 150 and 200kHz. In addition, in every high frequency excitation, there will be three forms of modes representing the low frequency excitation. Additionally, data obtained from the power spectrum using nonlinear vibroacoustic testing were used to plot the high frequencies of 60, 150, and 200 kHz with the laser measurement range.



Fig. 9. High frequency 60kHz





Figure 9 shows that the modulation intensity generally increases with the excitation level and decreases with increasing range. However, the largest R value is obtained in the 3rd vibration mode and the lowest in the 1st vibration mode. Interesting observations can be made when the R value decreases in the measurement range. The SLDV laser pointer shows the main factors contributing to vibration sound modulation. These results indicate that a high frequency level of 60 kHz can have a significant impact on sideband intensity. As seen in the graph, the trend of mode 3 clearly shows that the sideband intensity R value is higher compared to mode 2 and mode 1. The sideband intensity of mode 1 decreases due to the increase in range. laser vibration meter. In short, laser sensing decreases as range increases. Figure 10 shows that experimental testing at a high frequency of 150 kHz provides the most accurate data, better than other selected high frequencies. All vibration modes show constant trend R values from 1 meter to 3 meters respectively. Then the whole vibration mode increases slightly and then decreases sharply when it reaches 5 meters. Based on the overall illustration, it can be seen that the 3<sup>rd</sup> vibration mode has a higher R value than the  $2^{nd}$  and  $1^{st}$  vibration modes. The higher natural frequency, namely 150 kHz, also has a significant influence on the sideband intensity. The trend clearly shows that the R value is affected within a small range. In comparison, the high frequency of 150 kHz decreases as the laser range increases. However, the result is that the R value becomes less sensitive at high laser vibrometer ranges.



Based on Figure 11, it can be seen that the low excitation frequencies, namely 62.5 Hz, 109.5 Hz and 188 Hz, have almost the same trend as the other high frequencies. The trend shows that the sideband intensity corresponding to the R value will tend to be constant at the top of the range and will decrease slightly as the range increases. In short, laser distance measurements can affect the data due to increasing laser distance. As previously estimated, for the high frequency of 200 kHz, the R value decreases sharply after increasing the operating range of the laser vibrometer. However, the type of relationship changes significantly with the frequency of the selected mode. For all high excitation oscillations, the initial range clearly shows that the high frequency excitation is unaffected in a small range. This means that to capture the effectiveness of laser distance measurements on cracks in aluminum plates, longrange laser distance data is needed. Additionally, it can be difficult to differentiate laser effects based on R values because most have similar range values. Therefore, the analysis of the 200 kHz test also shows that the sideband intensity is stable within a certain range and then the laser sensitivity decreases as the range of the laser vibrometer increases. In summary, the selected frequencies were used to observe the effect of laser distance measurements on cracks in aluminum panels. This frequency shows relatively good sensitivity for both small and large crack values. The research results show that the 60 kHz frequency is the best choice. Sensitive enough to detect small cracks and able to detect relatively large cracks well. It also shows how high frequencies are affected by laser distance measurements. In comparison, all selected frequencies have the same trend and this clearly shows that the intensity of the sideband response to high frequencies depends on the shape of the vibration modes present in each test. These results show that the vibration mode that is most affected is the first vibration mode because in each result vibration mode 1 is more dominant than the second and third vibration modes.

#### **5.** Conclusion

The vibroacoustic method can be used to detect cracks because it has high sensitivity compared to other methods. In addition, the results collected in this study focus on vibrational acoustic wave modulation used to detect cracks in aluminum. There are three vibration modes to choose from for low frequency stimulation. The aim of this research is to determine the effect of laser distance measurements on fatigue cracks. This research involves signal harmonic analysis and sideband modulation. The trend of the results shows that fatigue crack detection is influenced by laser distance measurements. Data obtained from the power spectrum using nonlinear vibroacoustic testing were used to plot the high frequencies of 60, 150, and 200 kHz with the laser measurement range. And the research results show that the 60 kHz frequency is the best choice and is sensitive enough to detect small cracks and is able to detect relatively large cracks well. Nonlinear vibroacoustic test analysis shows that all three modal forms are affected by the laser beam enhancement. However, the most affected low frequency excitation is the double bending mode.

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